9. Ressonâncias Não Lineares

PGF 5005 - Mecânica Clássica

web.if.usp.br/controle
(Referências principais: Reichl, 1992;
Walker-Ford, Phyical Review 1969)

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2024

Ressonâncias Não Lineares

Referência:

L. Reichl, The Transition to Chaos

Capítulo 2

2.4.1 Single-Resonance Hamiltonians

In terms of action-angle variables, a general single-resonance Hamiltonian can be written

$$H = H_0(J_1, J_2) + \epsilon V_{n_1,n_2}(J_1, J_2)\cos(n_1\theta_1 - n_2\theta_2) = E, \tag{2.4.1}$$

where $(J_1, J_2, \theta_1, \theta_2)$ are action-angle variables. This system has a second isolating integral

$$I = n_2 J_1 + n_1 J_2 = C_2, \tag{2.4.2}$$

where C_2 is a constant. It is easy to see that Eq. (2.4.2) is an isolating integral. Write Hamilton's equations of motion for J_1 and J_2,

$$\frac{dJ_1}{dt} = -\frac{\partial H}{\partial \theta_1} = n_1 \epsilon V_{n_1,n_2} \sin(n_1\theta_1 - n_2\theta_2) \tag{2.4.3}$$

and

$$\frac{dJ_2}{dt} = -\frac{\partial H}{\partial \theta_2} = -n_2 \epsilon V_{n_1,n_2} \sin(n_1\theta_1 - n_2\theta_2). \tag{2.4.4}$$

Using Eqs. (2.4.3) and (2.4.4), we find that

$$\frac{dI}{dt} = 0. \tag{2.4.5}$$

(2,2) Resonance

To see more clearly how a resonance works, let us consider the specific case of a (2,2) resonance. Following Walker and Ford, we write the Hamiltonian

$$H = H_0(J_1, J_2) + \alpha J_1 J_2 \cos(2\theta_1 - 2\theta_2) = E, \tag{2.4.6}$$

where

$$H_0(J_1, J_2) = J_1 + J_2 - J_1^2 - 3J_1 J_2 + J_2^2. \tag{2.4.7}$$

It is useful to make a transformation from action-angle variables $(J_1, J_2, \theta_1, \theta_2)$ to a new set of variables $(\mathcal{J}_1, \mathcal{J}_2, \Theta_1, \Theta_2)$ via the canonical transformation $\mathcal{J}_1 = J_1 + J_2 = I' = \frac{I}{2}$, $\mathcal{J}_2 = J_2$, $\Theta_1 = \theta_2$, and $\Theta_2 = \theta_2 - \theta_1$. The Hamiltonian then takes the form

$$\mathcal{H} = \mathcal{J}_1 - \mathcal{J}_1^2 - \mathcal{J}_1\mathcal{J}_2 + 3\mathcal{J}_2^2 + \alpha\mathcal{J}_2(\mathcal{J}_1 - \mathcal{J}_2)\cos(2\Theta_2) = E. \qquad (2.4.8)$$

Since $\mathcal{H}$ is independent of Θ_1, in this new coordinate system $\mathcal{J}_1$ is constant. Hamilton's equations in this coordinate system become

$$\frac{d\mathcal{J}_1}{dt} = 0, \qquad (2.4.9.a)$$

$$\frac{d\Theta_1}{dt} = 1 - 2\mathcal{J}_1 - \mathcal{J}_2 + \alpha\mathcal{J}_2\cos(2\Theta_2), \qquad (2.4.9.b)$$

and

$$\frac{d\mathcal{J}_2}{dt} = 2\alpha\mathcal{J}_2\sin(2\Theta_2)(I' - \mathcal{J}_2), \qquad (2.4.10.a)$$

$$\frac{d\Theta_2}{dt} = -I' + 6\mathcal{J}_2 + \alpha\cos(2\Theta_2)(I' - 2\mathcal{J}_2). \qquad (2.4.10.b)$$

Since $\mathcal{J}_1$ is constant, Eqs. (2.4.10) can be solved first for $\mathcal{J}_2(t)$ and $\Theta_2(t)$ and then substituted into Eq. (2.4.9.b) to obtain $\Theta_1(t)$.

Let us now find the fixed points of these equations. The fixed points are points for which $\frac{d\mathcal{J}_2}{dt} = 0$ and $\frac{d\Theta_2}{dt} = 0$. Fixed points occur when $\Theta_2 = \frac{n\pi}{2}$ and $\mathcal{J}_2 = \mathcal{J}_o$, where $\mathcal{J}_o$ is a solution of the equation

$$-I' + 6\mathcal{J}_o + \alpha\cos(n\pi)(I' - 2\mathcal{J}_o) = 0. \tag{2.4.11}$$

Note that for $\alpha \ll 1$, $\mathcal{J}_o \approx \frac{I'}{6}$.

For *very small* α, the fixed points occur for $\mathcal{J}_2 = \mathcal{J}_o \approx \frac{I'}{6}$ and therefore for $J_1 \approx \frac{5I'}{6}$ and $J_2 \approx \frac{I'}{6}$. We can also find the range of energies for which these fixed points exist. Plugging $J_1 = 5J_2$ into Eq. (2.4.6), we find $J_1^2 - \frac{10J_1}{13} + \frac{25E}{39} = 0$ or $J_1 = \frac{5}{13}(1 \pm (1 - \frac{13E}{3})^{\frac{1}{2}}) = 5J_2$. Thus, the fixed points only exist for $E < \frac{3}{13}$ for very small α. For $E > \frac{3}{13}$, J_1 is no longer real.

The nature of the fixed points can be determined by linearizing the equations of motion about points ($\mathcal{J}_2 = \mathcal{J}_o, \Theta_2 = \frac{n\pi}{2}$). We let $\mathcal{J}_2(t) = \mathcal{J}_o + \Delta\mathcal{J}(t)$ and $\Theta_2(t) = \frac{n\pi}{2} + \Delta\Theta(t)$ and linearize in $\Delta\mathcal{J}(t)$ and $\Delta\Theta(t)$. We find

$$\frac{d}{dt}\begin{pmatrix} \Delta\mathcal{J}(t) \\ \Delta\Theta(t) \end{pmatrix} = \begin{pmatrix} 0 & 4\alpha\cos(n\pi)\mathcal{J}_o(I' - \mathcal{J}_o) \\ (6 - 2\alpha\cos(n\pi)) & 0 \end{pmatrix} \times \begin{pmatrix} \Delta\mathcal{J}(t) \\ \Delta\Theta(t) \end{pmatrix}. \tag{2.4.12}$$

The solution $\binom{\Delta\mathcal{J}(t)}{\Delta\Theta(t)}$ to Eq. (2.4.12) determines the manner in which trajectories flow *in the neighborhood of the fixed points.* For $\alpha \ll 1$ (and therefore $\mathcal{J}_o \approx \frac{I'}{6}$), these equations reduce to

$$\frac{d}{dt}\begin{pmatrix} \Delta\mathcal{J}(t) \\ \Delta\Theta(t) \end{pmatrix} \approx \begin{pmatrix} 0 & \frac{20\alpha I'^2}{36}\cos(n\pi) \\ 6 & 0 \end{pmatrix} \begin{pmatrix} \Delta\mathcal{J}(t) \\ \Delta\Theta(t) \end{pmatrix}. \tag{2.4.13}$$

Let us assume that Eq. (2.4.13) has a solution of the form

$$\begin{pmatrix} \Delta\mathcal{J}(t) \\ \Delta\Theta(t) \end{pmatrix} = e^{\lambda t} \begin{pmatrix} A_{\mathcal{J}} \\ A_{\Theta} \end{pmatrix}, \tag{2.4.14}$$

where $A_{\mathcal{J}}$ and A_{Θ} are independent of time. Then we can solve the resulting eigenvalue equation

$$\lambda \begin{pmatrix} A_{\mathcal{J}} \\ A_{\Theta} \end{pmatrix} = \begin{pmatrix} 0 & \frac{20\alpha I'^2}{36}\cos(n\pi) \\ 6 & 0 \end{pmatrix} \begin{pmatrix} A_{\mathcal{J}} \\ A_{\Theta} \end{pmatrix}$$

for both λ and $\begin{pmatrix} A_{\mathcal{J}} \\ A_{\Theta} \end{pmatrix}$. The eigenvalues are given by

$$\lambda_{\pm} = \pm \left(\frac{20\alpha I'^2 \cos(n\pi)}{6} \right)^{\frac{1}{2}},$$

and the solution to Eq. (2.4.13) can be written

$$\begin{pmatrix} \Delta\mathcal{J}(t) \\ \Delta\Theta(t) \end{pmatrix} = e^{\lambda_+ t}\, A_+ \begin{pmatrix} \frac{b}{\lambda_+} \\ 1 \end{pmatrix} + e^{\lambda_- t}\, A_- \begin{pmatrix} \frac{b}{\lambda_-} \\ 1 \end{pmatrix}, \tag{2.4.15}$$

where $b = \frac{20\alpha I'^2}{36}$, and A_+ and A_- are determined by the initial conditions. For n even, λ is real and the solutions contain exponentially growing and decreasing components, while for n odd, λ is pure imaginary and the solutions are oscillatory. For n even, the fixed points are hyperbolic (trajectories approach or recede from the fixed point exponentially), while for n odd, the fixed points are elliptic (trajectories oscillate about the fixed point).

A plot of some of the trajectories on the energy surface, $E = 0.18$, for coupling constant $\alpha = 0.1$, is given in Fig. 2.4.1. In this plot, we have transformed from polar coordinates $(\mathcal{J}_2, \Theta_2)$ to Cartesian coordinates (p, q) via the canonical transformation $p = -(2\mathcal{J}_2)^{\frac{1}{2}} \sin(\Theta_2)$ and $q = (2\mathcal{J}_2)^{\frac{1}{2}} \cos(\Theta_2)$. The elliptic and hyperbolic fixed points and the separatrix associated with them can be seen clearly. The region inside and in the immediate neighbor-

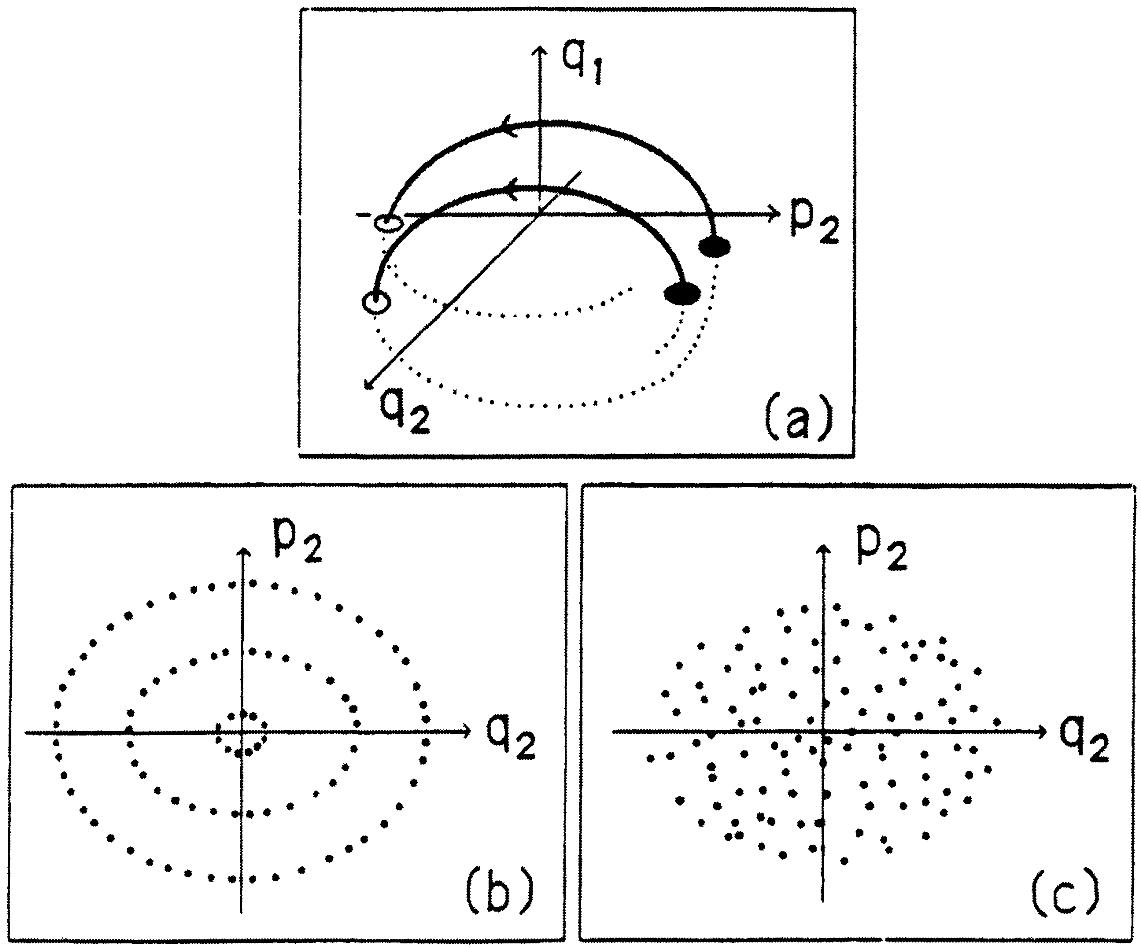


Figure 2.3.1. A Poincaré surface of section for a two degree of freedom system provides a two-dimensional map. (a) A surface of section may be obtained, for example, by plotting a point each time the trajectory passes through the plane $q_1 = 0$ with $p_1 \geq 0$. (b) If two isolating integrals exist, the trajectory will lie along one-dimensional curves in the two-dimensional surface. (c) If only one isolating integral exists (the energy), the trajectory will spread over a two-dimensional region whose extent is limited by energy conservation.

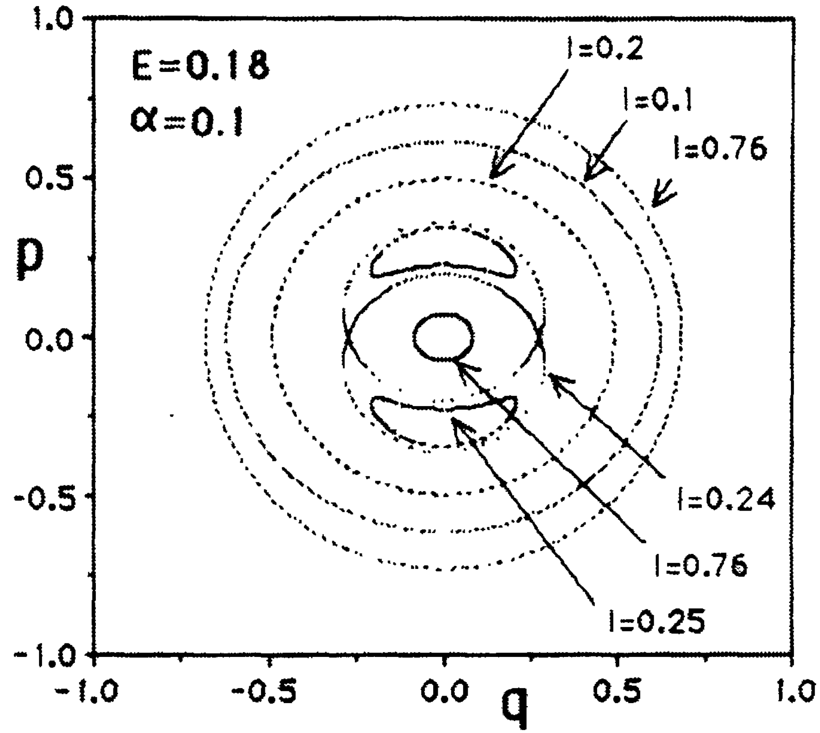


Figure 2.4.1. Phase space trajectories for the (2,2) resonance Hamiltonian in Eq. (2.4.8) ($p = -(2\mathcal{J}_2)^{\frac{1}{2}}\sin(\Theta_2)$ and $q = (2\mathcal{J}_2)^{\frac{1}{2}}\cos(\Theta_2)$). For all curves, $E = 0.18$ and $\alpha = 0.1$. The curves consist of discrete points because we have plotted points along the trajectories at discrete times.

Hamiltoniana Perturbada

$$H = H_0(J_1, J_2) + \epsilon V(J_1, J_2, \theta_1, \theta_2) \qquad \epsilon \ll 1$$

Soluções não perturbadas são periódicas; expandindo em série de Fourier:

$$H = H_0(J_1, J_2) + \epsilon \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} V_{n_1,n_2}(J_1, J_2) \cos(n_1\theta_1 + n_2\theta_2)$$

Antigas variáveis de ângulo e ação de $(J_1, J_2, \theta_1, \theta_2)$

Novas variáveis de ângulo e ação de H $(\mathcal{J}_1, \mathcal{J}_2, \Theta_1, \Theta_2)$

Função geratriz da Transformação Canônica

$$G(\mathcal{J}_1, \mathcal{J}_2, \theta_1, \theta_2) = \mathcal{J}_1\theta_1 + \mathcal{J}_2\theta_2 + \epsilon \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} g_{n_1,n_2}(\mathcal{J}_1, \mathcal{J}_2) \sin(n_1\theta_1 + n_2\theta_2)$$

Vamos determinar g_{n_1,n_2}

4 relações entre as antigas e as novas variáveis

$$J_i = \frac{\partial G}{\partial \theta_i} = \mathcal{J}_i + \epsilon \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} n_i g_{n_1,n_2} \cos(n_1\theta_1 + n_2\theta_2)$$

$$\Theta_i = \frac{\partial G}{\partial \mathcal{J}_i} = \theta_i + \epsilon \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} \frac{\partial g_{n_1,n_2}}{\partial \mathcal{J}_i} \sin(n_1\theta_1 + n_2\theta_2)$$

Substituimos θ_i e J_i na equação a seguir

$$H = H_0(J_1, J_2) + \epsilon \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} V_{n_1,n_2}(J_1, J_2) \cos(n_1\theta_1 + n_2\theta_2)$$

$$H'(\mathcal{J}_1, \mathcal{J}_2, \Theta_1, \Theta_2)$$
$$= H_0'(\mathcal{J}_1, \mathcal{J}_2) + \epsilon \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} (n_1\omega_1 + n_2\omega_2) g_{n_1,n_2} \cos(n_1\Theta_1 + n_2\Theta_2)$$
$$+ \epsilon \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} V_{n_1,n_2}(\mathcal{J}_1, \mathcal{J}_2) \cos(n_1\Theta_1 + n_2\Theta_2) + O(\epsilon^2)$$

$$\omega_i = \frac{\partial H_o'}{\partial \mathcal{J}_i}$$

Escolhendo

$$g_{n_1,n_2} = -\frac{V_{n_1,n_2}(\mathcal{J}_1, \mathcal{J}_2)}{(n_1\omega_1 + n_2\omega_2)}$$

obtemos

$$H'(\mathcal{J}_1, \mathcal{J}_2, \Theta_1, \Theta_2) = H_o'(\mathcal{J}_1, \mathcal{J}_2) + O(\epsilon^2)$$

As novas ações podem ser obtidas de

$$J_i = \mathcal{J}_i - \epsilon \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} \frac{n_i V_{n_1,n_2} \cos(n_1\Theta_1 + n_2\Theta_2)}{(n_1\omega_1 + n_2\omega_2)} + O(\epsilon^2)$$

A transformação encontrada para as variáveis existe se

$$|n_1\omega_1 + n_2\omega_2| \gg \epsilon V_{n_1,n_2}$$

A geratriz diverge se a órbita estiver na região de ressonância no espaço de fase

Resultado de validade geral, comum em vários sistemas!

to new variables $(\mathcal{I}_1, \mathcal{I}_2, \phi_1, \phi_2)$ via a canonical transformation given by the generating function

$$G(\mathcal{I}_1, \mathcal{I}_2, \phi_1, \phi_2) = \mathcal{I}_1\theta_1 + \mathcal{I}_2\theta_2 + \alpha g_{2,2}(\mathcal{I}_1, \mathcal{I}_2)\sin(2\theta_1 - 2\theta_2). \qquad (2.4.16)$$

Following the procedure outlined in Sect. 2.2, we find that $g_{2,2} = \frac{-\mathcal{I}_1\mathcal{I}_2}{(2\omega_1 - 2\omega_2)}$, where $\omega_1 = 1 - 2\mathcal{I}_1 - 3\mathcal{I}_2$ and $\omega_2 = 1 - 3\mathcal{I}_1 + 2\mathcal{I}_2$. The Hamiltonian to order α^2 is $H = H_o(\mathcal{I}_1, \mathcal{I}_2) + O(\alpha^2)$ and the action variables (neglecting terms of order α^2) are

$$J_1(t) = \mathcal{I}_1 - \frac{2\alpha\mathcal{I}_1\mathcal{I}_2\cos(2\omega_1 t - 2\omega_2 t)}{(2\omega_1 - 2\omega_2)} \qquad (2.4.17)$$

and

$$J_2(t) = \mathcal{I}_2 + \frac{2\alpha\mathcal{I}_1\mathcal{I}_2\cos(2\omega_1 t - 2\omega_2 t)}{(2\omega_1 - 2\omega_2)}. \qquad (2.4.18)$$

In order for these equations to have meaning, the following condition must hold:

$$|2\omega_1 - 2\omega_2| = |2\mathcal{I}_1 - 10\mathcal{I}_2| \gg 2\alpha\mathcal{I}_1\mathcal{I}_2.$$

However, near a resonance, $\mathcal{I}_1 \approx 5\mathcal{I}_2$. Therefore this condition breaks down in the neighborhood of a resonance zone. Actually this is to be expected since the resonance introduces a topological change in the flow pattern in the phase space.

(2,3) Resonance

Walker and Ford also studied a (2,3) resonance with Hamiltonian

$$H = H_o(J_1, J_2) + \beta J_1 J_2^{\frac{3}{2}} \cos(2\theta_1 - 3\theta_2) = E. \tag{2.4.19}$$

This again is integrable and has two isolating integrals of the motion, the Hamiltonian, H, and

$$I = 3J_1 + 2J_2 = C_3. \tag{2.4.20}$$

We can again make a canonical transformation, $J_1 = \mathcal{J}_1 - \frac{2}{3}\mathcal{J}_2, J_2 = \mathcal{J}_2, \theta_1 = \Theta_1, \theta_2 = \Theta_2 + \frac{2}{3}\Theta_1$ (note that $I = 3\mathcal{J}_1$). The Hamiltonian then takes the form

$$\mathcal{H} = \mathcal{J}_1 - \mathcal{J}_1^2 + \frac{\mathcal{J}_2}{3} - \frac{5\mathcal{J}_1\mathcal{J}_2}{3} + \frac{23}{9}\mathcal{J}_2^2 + \frac{\beta}{3}\mathcal{J}_2^{\frac{3}{2}}(3\mathcal{J}_1 - 2\mathcal{J}_2)\cos(3\Theta_2) = E \tag{2.4.21}$$

and the coordinate $\mathcal{J}_1$ is a constant of the motion since $\mathcal{H}$ is independent of Θ_1. The equations of motion for $\mathcal{J}_2$ and Θ_2 are

$$\frac{d\mathcal{J}_2}{dt} = \beta\mathcal{J}_2^{\frac{3}{2}}(3\mathcal{J}_1 - 2\mathcal{J}_2)\sin(3\Theta_2) \tag{2.4.22}$$

and

$$\frac{d\Theta_2}{dt} = \frac{1}{3} - \frac{5\mathcal{J}_1}{3} + \frac{46\mathcal{J}_2}{9} + \beta\mathcal{J}_2^{\frac{1}{2}}\left(\frac{3}{2}\mathcal{J}_1 - \frac{5}{3}\mathcal{J}_2\right)\cos(3\Theta_2). \tag{2.4.23}$$

It is easy to see that the fixed points occur for $\Theta_2 = \frac{n\pi}{3}$ and $\mathcal{J}_2 = \mathcal{J}_o$ where $\mathcal{J}_o$ satisfies the equation

$$\frac{1}{3} - \frac{5I}{9} + \frac{46\mathcal{J}_o}{9} + \beta\mathcal{J}_o^{\frac{1}{2}}\left(\frac{I}{2} - \frac{5}{3}\mathcal{J}_o\right)\cos(n\pi) = 0. \tag{2.4.24}$$

Pontos fixos
Para
$E_{2/3} > 0.16$

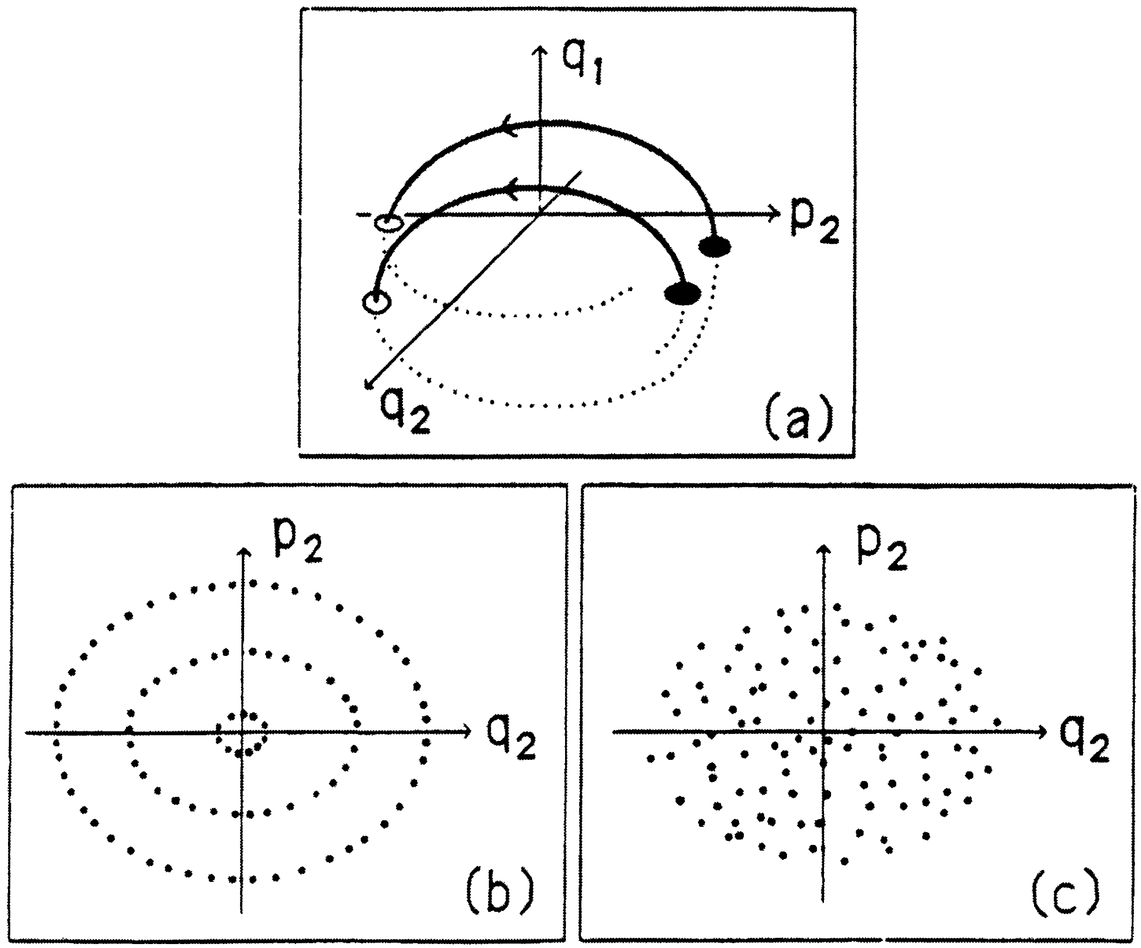
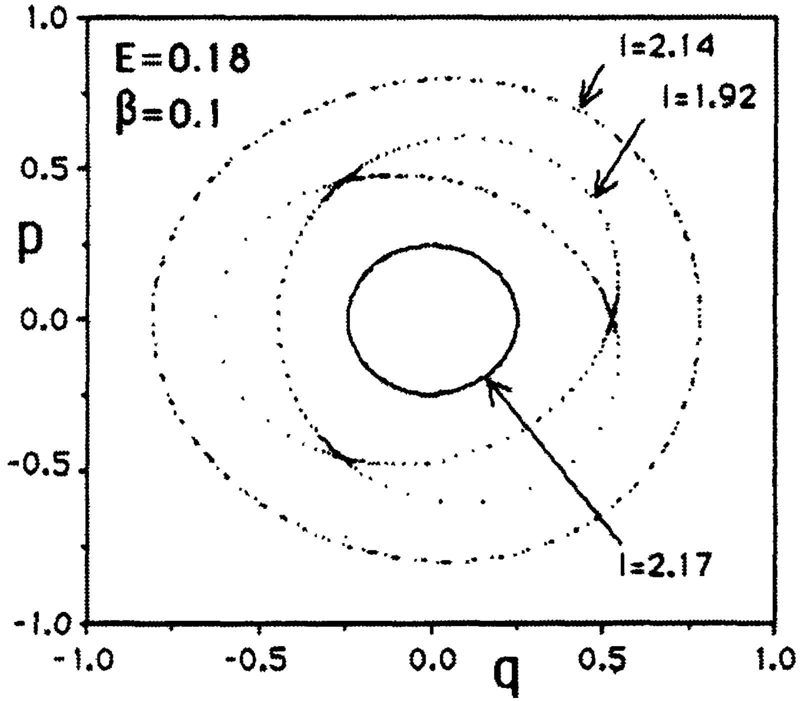


Figure 2.3.1. A Poincaré surface of section for a two degree of freedom system provides a two-dimensional map. (a) A surface of section may be obtained, for example, by plotting a point each time the trajectory passes through the plane $q_1 = 0$ with $p_1 \geq 0$. (b) If two isolating integrals exist, the trajectory will lie along one-dimensional curves in the two-dimensional surface. (c) If only one isolating integral exists (the energy), the trajectory will spread over a two-dimensional region whose extent is limited by energy conservation.



Pontos fixos para $E_{2/3} > 0.16$

Figure 2.4.2. A plot of some phase space trajectories obtained for the (2,3) resonance Hamiltonian in Eq. (2.4.19). All curves have energy $E = 0.18$ and coupling constant $\beta = 0.1$ but have different values of the constant of motion, I. The three hyperbolic and three elliptic fixed points as well as the separatrix of the (2,3) resonance are clearly seen. The curves consist of discrete points because we plot points along the trajectories at discrete times. We have set $p = -(2\mathcal{J}_2)^{\frac{1}{2}} \sin(\Theta_2)$ and $q = (2\mathcal{J}_2)^{\frac{1}{2}} \cos(\Theta_2)$.

of the system are varied. Walker and Ford show this for the Hamiltonian with two primary resonances,

$$\begin{aligned} H = H_o(J_1, J_2) &+ \alpha J_1 J_2 \cos(2\theta_1 - 2\theta_2) \\ &+ \beta J_1 J_2^{\frac{3}{2}} \cos(2\theta_1 - 3\theta_2) = E. \end{aligned} \tag{2.4.25}$$

The surface of section for this Hamiltonian is shown in Fig. 2.4.3.

Hamilton's equations for the two-resonance system can be written

$$\begin{aligned} \frac{dJ_1}{dt} = -\frac{\partial H}{\partial \theta_1} &= 2\alpha J_1 J_2 \sin(2\theta_1 - 2\theta_2) \\ &+ 2\beta J_1 J_2^{\frac{3}{2}} \sin(2\theta_1 - 3\theta_2), \end{aligned} \tag{2.4.26}$$

$$\begin{aligned} \frac{dJ_2}{dt} = -\frac{\partial H}{\partial \theta_2} &= -2\alpha J_1 J_2 \sin(2\theta_1 - 2\theta_2) \\ &- 3\beta J_1 J_2^{\frac{3}{2}} \sin(2\theta_1 - 3\theta_2), \end{aligned} \tag{2.4.27}$$

$$\begin{aligned} \frac{d\theta_1}{dt} = \frac{\partial H}{\partial J_1} &= 1 - 2J_1 - 3J_2 + \alpha J_2 \cos(2\theta_1 - 2\theta_2) \\ &+ \beta J_2^{\frac{3}{2}} \cos(2\theta_1 - 3\theta_2), \end{aligned} \tag{2.4.28}$$

$$\begin{aligned} \frac{d\theta_2}{dt} = \frac{\partial H}{\partial J_2} &= 1 - 3J_1 + 2J_2 + \alpha J_1 \cos(2\theta_1 - 2\theta_2) \\ &+ \frac{3}{2}\beta J_1 J_2^{\frac{1}{2}} \cos(2\theta_1 - 3\theta_2). \end{aligned} \tag{2.4.29}$$

Constantes de Movimento

$F_1 = 2J_1 + 2J_2$
$F_2 = 3J_1 + 2J_2$

Figura 2 - Funções H, F_1 e F_2 em função do tempo
($\alpha = 0.1$, $\beta = 0$ e $E = 0.18$)

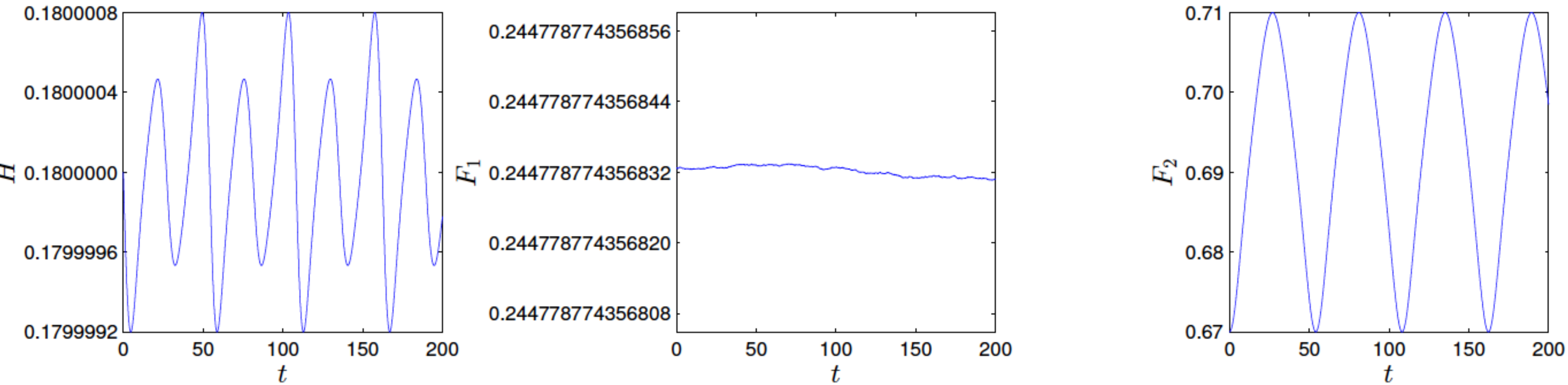
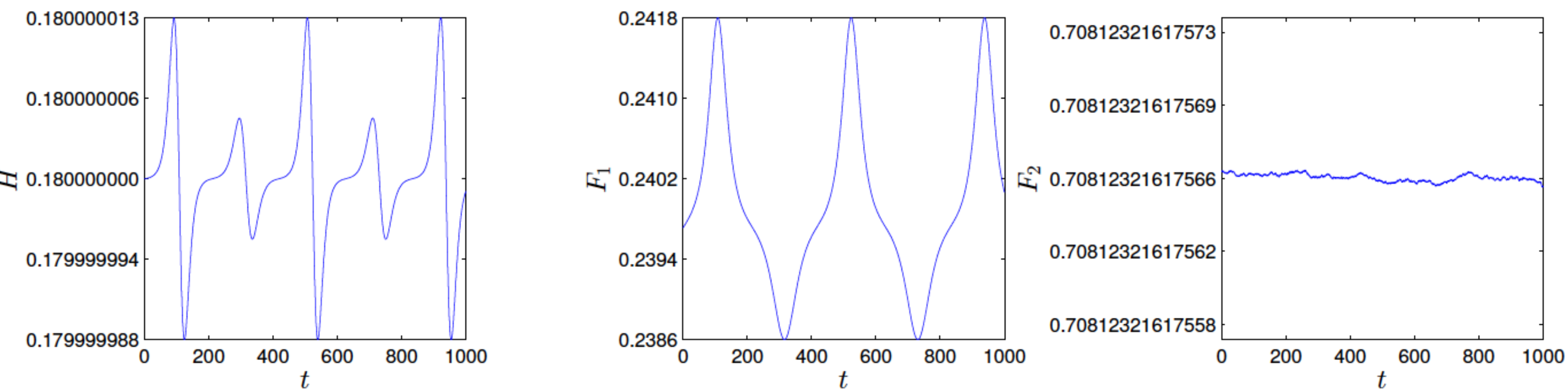
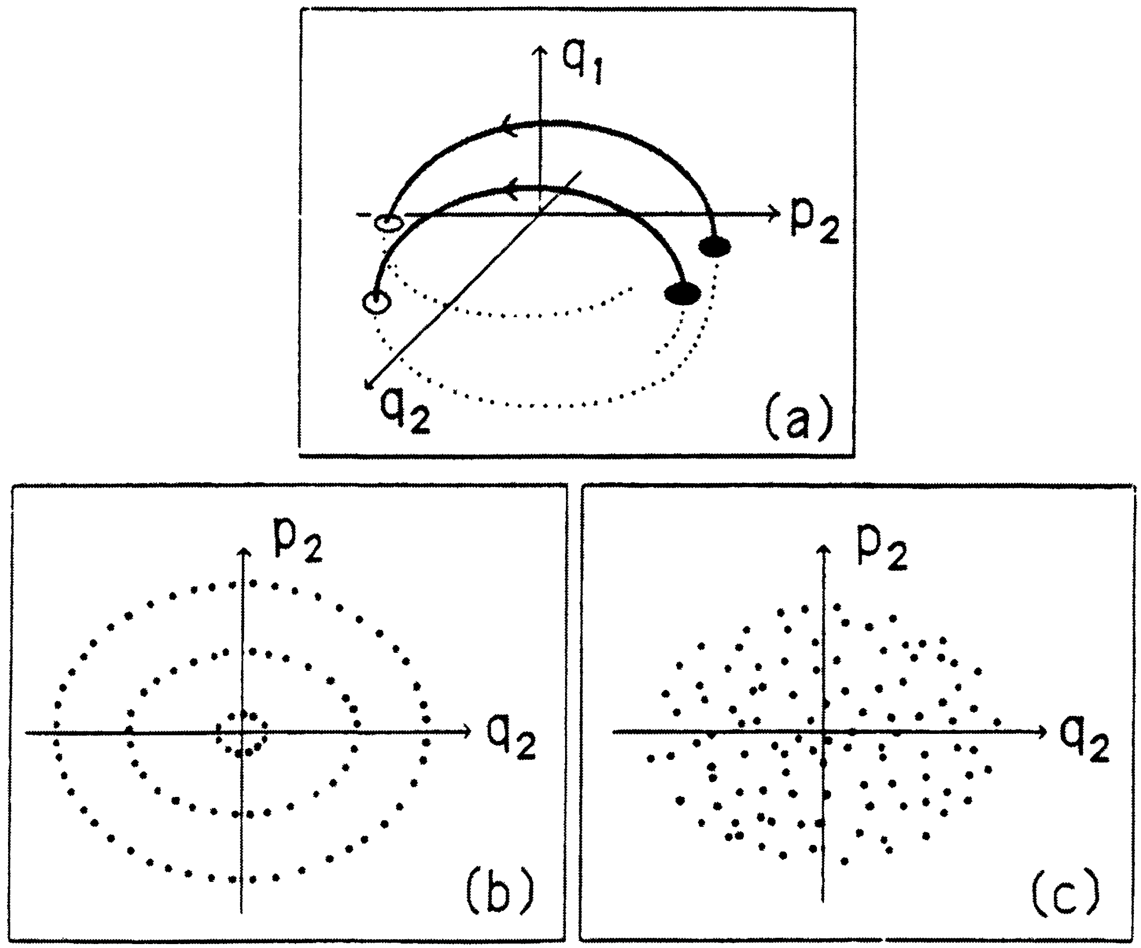


Figura 4 - Funções H, F_1 e F_2 em função do tempo
($\alpha = 0$, $\beta = 0.1$ e $E = 0.18$)





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Ressonância 2/2

$$H = H_0(J_1, J_2) + \alpha J_1 J_2 \cos(2\varphi_1 - 2\varphi_2)$$

$I = J_1' + J_2$ H, I constantes de movimento H –integrável

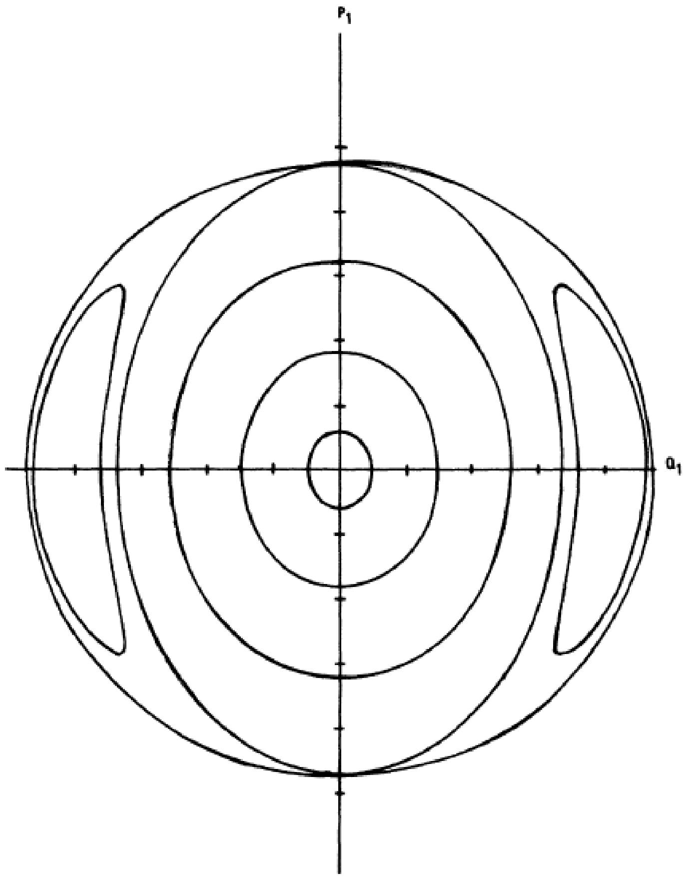


FIG. 6. Typical level curves for an isolated, 2-2 resonance computed algebraically.

Curvas de nível com I cte.

Ilhas surgem para E=0.
Aumentando E, ilhas se afastam do centro e aumentam sua largura.

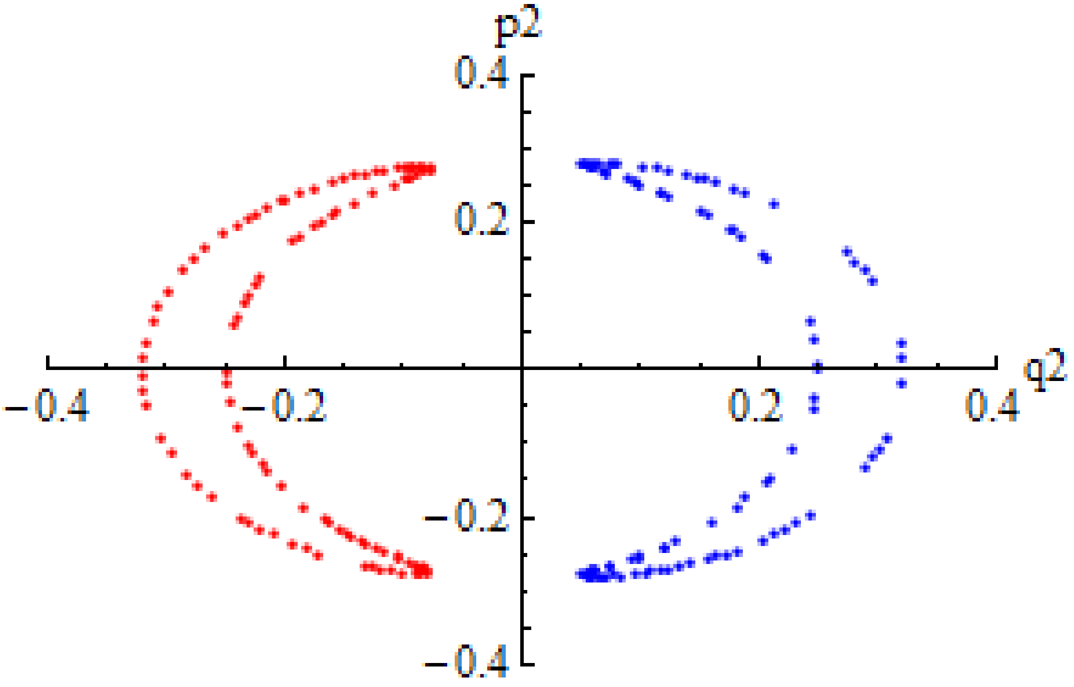
Pontos fixos
Para
$E_{2/2} < 3/13 = 0,23$

Duas Ilhas

Ressonância (2,2) no plano $p_2 \times q_2$

(equivale às ilhas de ressonância exibidas nas figuras 8-12 do artigo de Walker e Ford de 1969)

$$m = n = 2$$



Ressonância 2/3

$$H = H_0(J_1, J_2) + \beta J_1 J_2{}^{3/2} \cos(2\varphi_1 - 3\varphi_2)$$

H - integrável

$$I = 3J_1 + 2J_2$$

H, I constantes de movimento

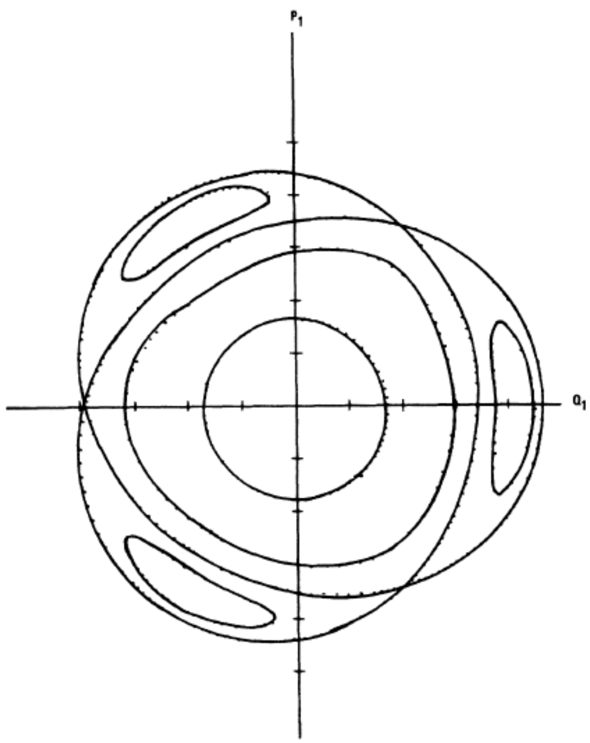


FIG. 7. Typical level curves for an isolated, 3-2 resonance computed algebraically. The dots represent points computed using Eq. (25); the curves were drawn in freehand. The chain of three islands first appears at the origin for $E=0.08$. All the widths of the islands including this one increase with increasing energy.

$$0{,}16 \leq E$$

Curvas de nível com I cte.

Ilhas surgem para E=0,16.
Aumentando E, ilhas se afastam do centro e aumentam sua largura.

Ressonância Dupla
2/2 e 2/3

$$H = H_0(J_1, J_2) + \alpha J_1 J_2 \cos(2\varphi_1 - 2\varphi_2)$$

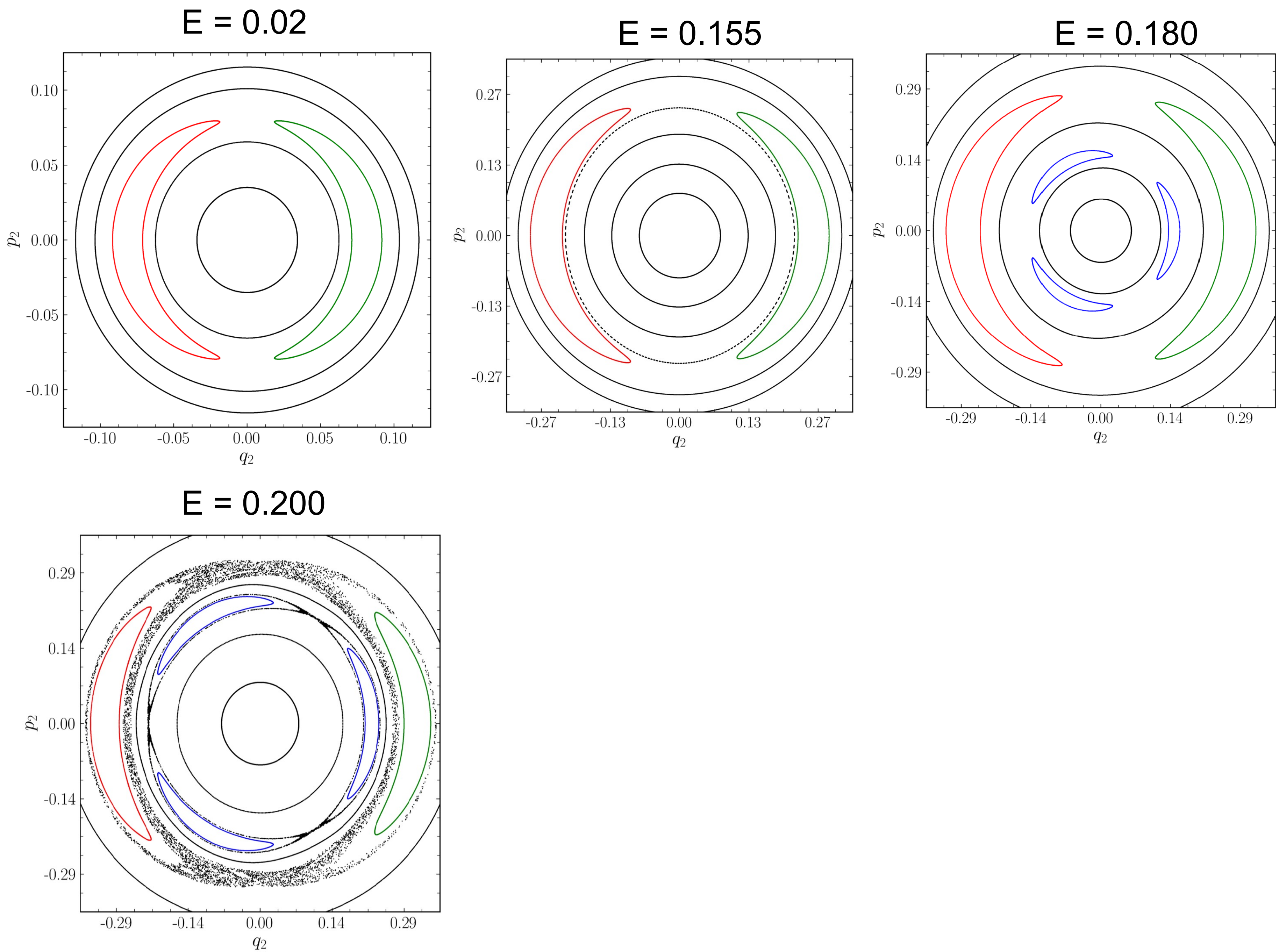
$$+ \beta J_1 J_2^{3/2} \cos(2\varphi_1 - 3\varphi_2),$$

$$\alpha = \beta = 0.02$$

Aumento da Região Caótica com E

E = 0,100

E =0,140


E = 0.02
E = 0.155
E = 0.180
E = 0.200
0.10
0.05
0.00
-0.05
-0.10
0.27
0.13
-0.13
-0.27
0.29
0.14
-0.14
-0.29
p_2
q_2